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The Effects of a Finite Pulse Time in the Flash Thermal Diffusivity Method

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After a brief review of the finite pulse time effects in flash thermal diffusivity measurements, an analytical expression for an exponential shape pulse was determined using the Green function method. The results were compared with those obtained by Larson and Koyama. It was found that, using the Larson and Koyama equation, when the dimensionless time ω is equal to zero, the dimensionless temperature rise V cannot reach zero, and when ω_{p}, the time characterizing the dimensionless pulse, approaches $1/n^2$ $(n = 1, 2, 3,...)$, a large error of $\omega_{1/2}$ will result. These contradictions have been resolved by the present work. In other respects, both sets of results concurred. The results are compared with the triangular pulse and are discussed.

KEY WORDS: finite pulse time effect; flash method; thermal diffusivity.

1. INTRODUCTION

The flash method for measuring thermal diffusivity was first proposed by Parker et al. [1] 30 years ago. Since the seventies, it has gained in popularity to such an extent that over 80% of the current thermal diffusivity measurements utilize this technique. This growth can be attributed to the basic simplicity of the method, the small sample size required, the rapidity of the measurements, the high reliability and accuracy, the ability to use the technique from cryogenic to very high temperatures, and the extensive adaptability for measuring materials whose diffusivities range from 10^{-7} to $10^{-3}\ \mathrm{m}^2 \cdot \mathrm{s}^{-1}$. A schematic diagram of one such apparatus is shown in Fig. 1 [2]. Very simply, the method involves subjecting the front surface of a small size sample to a short energy pulse

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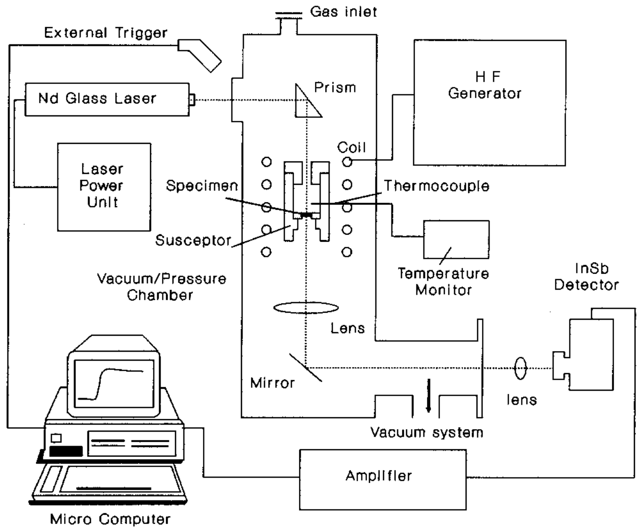

Fig. 1. Schematic diagram of UMIST laser flash apparatus.

and measuring the resultant temperature rise of the rear surface. The dimensionless temperature rise V can be expressed by [1]

$$V(L, t) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n \exp(-n^2 \omega) \tag{1}$$

where

$$V(L, t) = \frac{T(L, t)}{T_m} \tag{2}$$

$$\omega = \frac{\pi^2 \alpha t}{L^2} \tag{3}$$

and L is the sample thickness, t is time, and T_m is the maximum temperature rise of the rear surface, as shown in Fig. 2. The diffusivity is usually determined from the relation

$$\alpha = \frac{0.1388 L^2}{t_{1/2}} \tag{4}$$

where $t_{1/2}$ is the time from the initiation of the energy pulse to where the temperature rise on the rear surface has reached half of its maximum value, T_M. Equation (2) is based upon the duration of the energy pulse being short compared to $t_{1/2}$. If this is not the case, then the details of the shape and duration of the energy pulse become important and must be

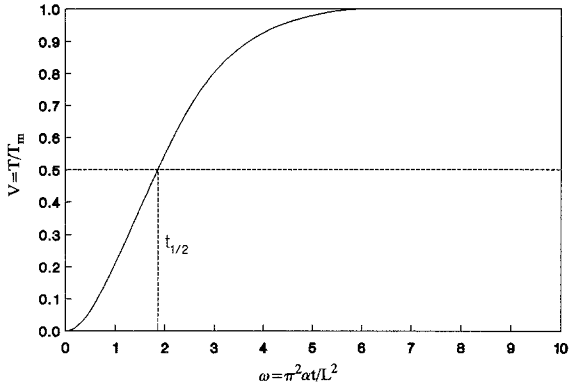

Fig. 2. Dimensionless plot of rear surface temperature history.

considered in deriving an appropriate expression for thermal diffusivity α. The shape of the energy pulse may vary depending on the energy source. Usually, a Nd glass laser is used as the source of energy pulse, but flash lamps [1], electron beams [3], and other types of lasers [4] have also been used. The shape and duration of the energy pulse affect the rear temperature response curve. This is known as the finite time pulse effect and causes the rear temperature history to lag behind in the curve in Fig. 2 [5]. During the last 20 years, many authors have derived various equations for different pulse shapes. Among them, Larson and Koyama [6] derived an equation for the case of an exponential-shape input pulse using contour integration; although a square wave is considered a closer approximation to the shape of the pulse from a Nd glass laser [5]. However, if an exponential pulse is assumed, some of the results obtained from the equation of Larson and Koyama are not acceptable. The purpose of the present paper was to derive an equation that can used to correct the finite pulse time effect for such exponential pulse shape to yield improved results.

2. ANALYSES

2.1. Equation of Larson and Koyama

For a pulse whose shape is described by $\phi(t)$, Larson and Koyama [6] define

$$\phi(t)=\frac{t}{t_p}\exp\left(1-\frac{t}{t_p}\right) \tag{5}$$

where t_p is the time from the initiation of the energy pulse till the maximum output of the energy pulse has been reached. Using the coordinate system given in Fig. 3 for a pulse incident upon the front surface and no heat losses from the sample, the dimensionless expression derived by Larson and Koyama is

$$V_1(L, t) = 1 - \frac{(\pi^2/\omega_p)^{1/2} \exp(-\omega/\omega_p)}{2 \sin(\pi^2/\omega_p)^{1/2}} \left[1 + 2\frac{\omega}{\omega_p} + \left(\frac{\pi^2}{\omega_p}\right)^{1/2} ctg(\pi^2/\omega_p)^{1/2} \right] + 2 \sum_{n=1}^{\infty} (-1)^n \frac{\exp(-n^2\omega)}{(1-n^2\omega_p)^2} \tag{6}$$

$$V_1(L, t) = \frac{T(L, t) - T_n}{T(L, \infty) - T_n} \tag{7}$$

$$\omega_p = \frac{\pi^2 \alpha t_p}{L^2} \tag{8}$$

$$\omega = \frac{\pi^2 \alpha t}{L^2} \tag{9}$$

where T_n is the ambient temperature.

2.2. Revised Mathematical Derivation

It is more convenient to select the coordinate as shown in Fig. 4, whence the initial and boundary conditions become

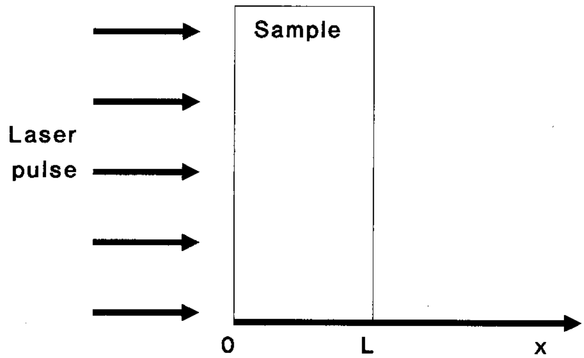

Fig. 3. Coordinate selected by Larson and Koyama [6].

$$\rho c_{\mathrm{p}} \frac{\partial v}{\partial t} - k \frac{\partial^2 v}{\partial x^2} = 0 \tag{10}$$

$$\left.\begin{aligned} -k \left.\frac{\partial v}{\partial x}\right|_{x=0} &= 0 \\ -k \left.\frac{\partial v}{\partial x}\right|_{x=L} &= -\frac{q_0}{t_{\mathrm{p}}^2} t e^{-t/t_{\mathrm{p}}} \\ v(x, 0) &= 0 \end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned} v(x, t) &= y(x, t) + z(x, t) \\ y(x, t) &= \frac{x^2 q_0}{2Lkt_{\mathrm{p}}^2} t \exp(-t/t_{\mathrm{p}}) \end{aligned}\right\} \tag{12}$$

It can be shown that Eq. (11) reduces to

$$\left.\begin{aligned} \frac{\partial z}{\partial t} - \alpha \frac{\partial^2 z}{\partial x^2} &= f(x, t) \\ \left.\frac{\partial z}{\partial x}\right|_{x=0} &= 0 \\ \left.\frac{\partial z}{\partial x}\right|_{x=L} &= 0 \\ z(x, 0) &= 0 \end{aligned}\right\} \tag{13}$$

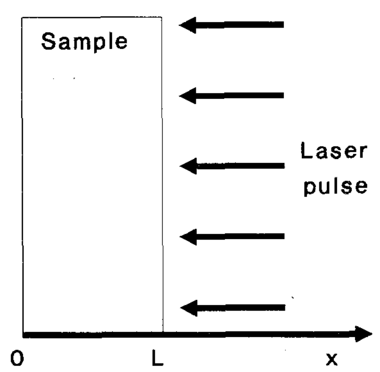

Fig. 4. Coordinate selected in the present paper.

The Green function of Eq. (13) satisfies the following conditions:

$$f(x, t)=\frac{-q_0}{Lkt_p^2}\left[\frac{x^2}{2}\left(1-\frac{t}{t_p}\right)+\alpha t\right]\exp(-t/t_p) \tag{14}$$

$$\left.\begin{aligned}&\frac{\partial G}{\partial t}-\alpha\frac{\partial^2 G}{\partial x^2}=0\\&\left.\frac{\partial G}{\partial x}\right|_{x=0}=0\\&\left.\frac{\partial G}{\partial x}\right|_{x=L}=0\\&G(x, t)|_{t=\tau+0}=\delta(x-\xi)\end{aligned}\right\} \tag{15}$$

The solution of Eq. (15) is

$$\left.\begin{aligned}&G(x, t; \xi, \tau)=\sum_{n=0}^{\infty} c_n(\xi, \tau)\exp\left[-\frac{n^2\pi^2\alpha(t-\tau)}{L^2}\right]\cos\frac{n\pi x}{L}\\&c_n(\xi, \tau)=\frac{2}{L\delta_n}\cos\frac{n\pi\xi}{L}\qquad (n=0, 1, 2;...)\end{aligned}\right\} \tag{16}$$

The solution of Eq. (13) can be written as

$$\left.\begin{aligned}z(x, t)&=\int_{\tau=0}^{t}\int_{\xi=0}^{L} f(\xi, \tau)\, G(x, t; \xi, \tau)\, d\tau\, d\xi\\&=\frac{-q_0}{Lkt_p^2}\frac{2}{L}\sum_{n=0}^{\infty}\frac{1}{\delta_n}\cos\left(\frac{n\pi x}{L}\right)\int_{\tau=0}^{t}\int_{\xi=0}^{L}\left[\frac{\xi^2}{2}\left(1-\frac{\tau}{t_p}\right)-\alpha\tau\right]\\&\quad\times\exp(-\tau/t_p)\cos\left(\frac{n\pi\xi}{L}\right)\exp\left[\frac{-n^2\pi^2\alpha(t-\tau)}{L^2}\right]d\tau\, d\xi\end{aligned}\right\} \tag{17}$$

Let

$$z_m=\frac{q_0}{LC_p\rho} \tag{18}$$

where z_m is the maximum temperature rise.

Therefore $z(x, t)$ is given by

$$z(x, t)=z_m\left\{1-\left(\frac{\pi^2\omega}{6\omega_p^2}+\frac{\omega}{\omega_p}+1\right)e^{-\omega/\omega_p}+2\sum_{n=1}^{\infty}(-1)^n\cos\frac{n\pi x}{L}\right.$$
$$\left.\times\left[\frac{e^{-n^2\omega}}{(1-n^2\omega_p)^2}-\left(\frac{1}{1-n^2\omega_p}+\frac{\omega}{n^2\omega_p^2}\right)\frac{e^{-\omega/\omega_p}}{1-n^2\omega_p}\right]\right\} \tag{19}$$

At the rear surface of the sample

$$x = 0, \qquad y(0, t) = 0$$

then

$$v(0, t) = y(0, t) + z(0, t) = z(0, t)$$

Let

$$V_2(0, t) = \frac{v(0, t)}{z_m} \tag{20}$$

The resulting equation can be written as

$$V_2(0, t) = 1 - \left(\frac{\pi^2 \omega}{6\omega_p^2} + \frac{\omega}{\omega_p} + 1\right) e^{-\omega/\omega_p} + 2 \sum_{n=1}^{\infty} (-1)^n \times \left[\frac{e^{-n^2\omega}}{(1 - n^2\omega_p)^2} - \left(\frac{1}{1 - n^2\omega_p} + \frac{\omega}{n^2\omega_p^2}\right) \frac{e^{-\omega/\omega_p}}{1 - n^2\omega_p}\right] \tag{21}$$

3. COMPARISON BETWEEN EQ. (6) AND EQ. (21)

The present equation [Eq. (21)] and that of Larson and Koyama [Eq. (6)] were computerized for various values of ω, ω_p, and V. Selected values of V_1 [from Eq. (6)] and V_2 [from Eq. (21)] are given in Table I.

It is shown that, when ω_p approaches $1/n^2$ ($n = 1, 2, 3, 4$) and $\omega = 0$, the dimensionless temperature rise $V_2 = 0$. However, at the same time, V_1

Table I. The Quantities V_1 and V_2 from Various Values of ω_p

	$\omega = 0$		$\omega = 0.5$		$\omega = 1.5$	
ω_p	V_1	V_2	V_1	V_2	V_1	V_2
1.005	8.1×10^{-5}	0	1.6×10^{-4}	1.4×10^{-4}	0.063	0.063
0.4	5.3×10^{-4}	0	8.9×10^{-4}	9.2×10^{-4}	0.21	0.21
0.252	1.4×10^{-3}	0	1.9×10^{-3}	2.1×10^{-3}	0.31	0.31
0.111	7.7×10^{-3}	0	6.5×10^{-3}	6.7×10^{-3}	0.45	0.45
0.063	2.7×10^{-2}	0	1.2×10^{-2}	1.2×10^{-2}	0.50	0.50
0.05	4.7×10^{-2}	0	1.5×10^{-2}	1.5×10^{-2}	0.51	0.51
0.041	7.8×10^{-2}	0	1.7×10^{-2}	1.7×10^{-2}	0.52	0.52
0.005	4.82×10^{3}	0	3.3×10^{-2}	3.3×10^{-2}	0.55	0.55
0.0024	6.347×10^{2}	0	3.5×10^{-2}	3.5×10^{-2}	0.56	0.56

Table II. Values of $\omega_{1/2}$ When ω_p Approaches $1/1^2$ and $V_1 = V_2 = 1/2$

ω_p	$\omega_{1/2}$	
	By Eq. (6)	By Eq. (21)
1.005	3.05	3.34
1.020	3.35	3.36
0.995	3.53	3.32
0.980	3.30	3.29

does not equal zero, but changes as ω_p varies. When ω_p is far from $1/n^2$, the values of V_1 are in agreement with V_2. If $\omega_p = 1/n^2$, both V_1 and V_2 are undefined.

If we let $V_1 = V_2 = \frac{1}{2}$, the corresponding values of $\omega_{1/2}$, obtained from Eqs. (6) and (21), may be deduced for various values of $1/n^2$. These are computed for $n = 1, 2, 4$ and listed in Tables II, III and IV.

From Tables I–IV, it is obvious that, when ω_p approaches $1/n^2$, there are significant errors in values from Eq. (6); for example, if $\omega_p = 1.005$, then $\omega_{1/2} = 3.05$; when $\omega_p = 0.995$, then $\omega_{1/2} = 3.53$. This means that, even though the values of ω_p change by only about 1%, that $\omega_{1/2}$ would be changed by more than 15%, whereas the corresponding values of $\omega_{1/2}$ from Eq. (21) would have a relative error of less than 1%.

4. COMPARISON WITH A TRIANGULAR PULSE

The shape of the exponential energy pulse can be approximated by a triangular pulse of duration τ with the maximum occurring at $\beta\tau$, where β

Table III. Values of $\omega_{1/2}$ When ω_p Approaches $1/2^2$ and $V_1 = V_2 = 1/2$

ω_p	$\omega_{1/2}$	
	By Eq. (6)	By Eq. (21)
1.25015	2.29	1.90
1.25020	1.92	1.90
0.24975	1.78	1.90
0.24970	1.88	1.90

Table IV. Values of $\omega_{1/2}$ When ω_p Approaches $1/4^2$ and $V_1 = V_2 = 1/2$

ω_p	$\omega_{1/2}$	
	By Eq. (6)	By Eq. (21)
1.25015	1.50	1.50
1.25020	1.50	1.50
0.24975	1.50	1.50
0.24970	1.50	1.50

is a fraction between zero and one. The equation for calculation of thermal diffusivity α_t for a triangular-shaped pulse of duration θ can be given as [5]

$$\alpha_t = \frac{c_1 L^2}{c_2 t_{1/2} - \tau} \tag{22}$$

where the constants c_1 and c_2 of Eq. (22) vary with β. For example, when $\beta = 0.15$, $c_1 = 0.34844$ and $c_2 = 2.5106$, but when $\beta = 0.28$, $c_1 = 0.31550$ and $c_2 = 2.2730$, and when $\beta = 0.5$, $c_1 = 0.27057$ and $c_2 = 1.9496$.

Equation (21) was solved for various values of $\omega_{1/2}$ and ω_p. Selected values are plotted in Fig. 5. Since the relation between $\omega_{1/2}$ and ω_p is nearly linear up to values for ω_p of 0.24, a closed linear solution can be obtained,

$$\omega_{1/2} = 2.13\omega_p + 1.365 \tag{23}$$

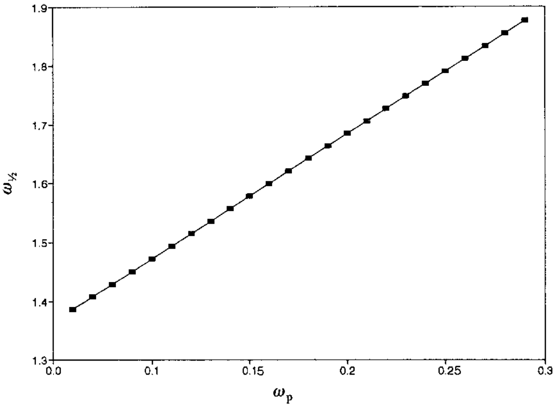

Fig. 5. Relationship between ω_p and $\omega_{1/2}$.

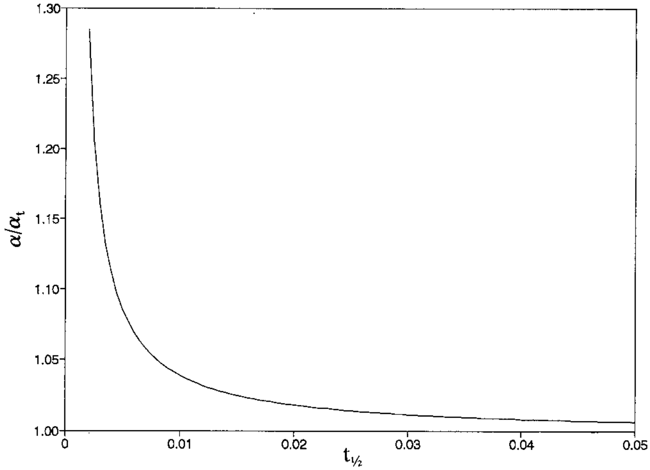

Fig. 6. Relationship between $t_{1/2}$ and α/α_t for $0 < t_{1/2} < 0.05$ s for $t_p = 6 \times 10^{-4}$ s.

Combining Eq. (23) with Eqs. (8) and (9) yields

$$\alpha = \frac{0.066L^2}{0.476t_{1/2} - t_p} \tag{24}$$

Using the equation for an assumed triangular-shaped pulse in the high-temperature thermal conductivity laboratory of UMIST, the following formula is derived for $\beta = 0.55$, $\tau = 6 \times 10^{-4}$ s:

$$\alpha_t = \frac{0.2613L^2}{1.882t_{1/2} - 6 \times 10^{-4}} \tag{25}$$

If we let $t_p = 0.55\tau$ in Eq. (24) and compare this with Eq. (25), the relation among α, α_t, and $t_{1/2}$ can be obtained. The results between α/α_t and $t_{1/2}$ are plotted in Fig. 6. It is shown that, under the same conditions, the values of thermal diffusivity calculated by Eq. (24) are always higher then those obtained using Eq. (25). The curve in Fig. 6 can be used to correct the thermal diffusivity results calculated using the triangular-shape approximation.

5. CONCLUSION

When $\omega \to \infty$ and $\varepsilon_p = 0$, the dimensionless temperature rise V_1 using the Larson and Koyama equation [Eq. (6)] and that obtained in the present analysis [Eq. (21)] both reduce to the equation given by Parker et al. [1]. When $\omega = 0$, the dimensionless temperature rise V_2 calculated by the present equation [Eq. (21)] is equal to zero, but for the Larson and Koyama equation [Eq. (6)], V_1 is not equal to zero, thus violating its initial boundary condition. When ω_p approaches $1/n^2$, the error in V_1 calculated by Eq. (6) is much larger than that in V_2 of Eq. (21). The results calculated by both equations are in good agreement where the dimensionless pulse characterization time ω_p deviates from $1/n^2$. When an exponential pulse is approximated by a triangular-shape pulse, the values of thermal diffusivity are always lower than what they should be and can easily be corrected according to Fig. 6.

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